ALAGU JOTHI ACADEMY

(An International Standard CBSE Sr. Sec. School)

WORK SHEET 2017– 18

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Name: ___________________ **MATHEMATICS** CHAPTERS:1,2,3,4,13

CHAPTER 1

SHORT ANSWER TYPE:

1. Let T be the set of all triangles drawn in a plane , prove that the following relations on T are equivalence relations (i) HAVE EQUAL AREA (ii) HAVE EQUAL PARAMETER
2. Let N be the set of all natural numbers and R be the relation on N x N defined by (a, b) R (c, d) If ad(b+ c)= bc (a+ d) . check whether R is an equivalence relation
3. Show that the relation R in the set {1,2,3} given by R={(1,2) , (2,1)} is symmetric but neither reflexive nor transitive .
4. Show that the relation R in the set A of all the books in a library of a college, given by R={(x, y) : x and y have same number of pages} is an equivalence relation .
5. Show that the relation R in the set A={1,2,3,4,5,6,7} given by R= { (a, b) : | a-b | is even }, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2,4} are related to each other . But no element of {1,3,5,7} is related to any element of {2,4,6}
6. Show that each of the relation R in the set A={x∈ W: 0≤x≤10}, given by (i) R= { (a, b) :| a-b |is a multiple of 3} . Find the set of all elements related to 3. Find the set of all elements related to 3 in each case.
7. Show that the relation R in the set A of points in a plane given by R = {(P,Q) : distance of the point P from the origin is same as the distance of the point Q from the origin} , is equivalence relation. Further , show that the set of all points related to a point P≠(0,0) is the circle passing through P with origin as centre .
8. Show that the relation R defined in the set A of all triangles as R={ (T1,T2) : T1 is similar to T2},is equivalence relation . Consider three right angle triangles T1 with sides 3,4,5 T2 with sides 5,12,13 and T3 with sides 6,8,10. Which triangles among T1,T2 and T3 are related ?
9. Show that the relation R defined in the set A of all polygons as R= { (P1,P2) : P1 and P2 have same number of sides }, is an equivalence relation . What is the set of all elements in A related to the right angle triangle T with sides 3,4and 5?
10. Let L be the set of all lines in XY plane R be the relation in L defined as R= { (L1,L2) : L1is parallel to L2 }. show that R is an equivalence relation . Find the set of all lines related to the lines y=2x+4 .
11. If R1 and R2 are equivalence relations in a set A, show that R1 ∩ R2 is also an equivalence relation.
12. Let R be a relation on the set A of ordered pairs of positive integers defined by (x,y)R(u ,v) if and only if xv=y u . Show that R is an equivalence relation.

LONG ANSWER TYPE:

1. Determine whether the relation R on the set R of all the real numbers as R={(a, b):a, b∈R and a-b+$\sqrt{3}$ ∈ S, Where S is the set of all the irrational numbers}, is reflexive, symmetric and transitive.
2. Consider f:R_+→ [-9,∞] given by f(x)=5x^2+6x-9. Prove that f is invertible with f^{-1}(y)=$\left(\left(\sqrt{(54+5y}\right)-3/5\right)$.
3. Let A={-1,0,1,2}, B={-4,-2,0,2} and f, g:A→B be functions defined by f(x) =x^2-x, x∈A and g(x)=2|x-$\frac{1}{2}$|-1, x∈A , Find gof (x) and hence show that f=g= gof . (6)
4. Show that the function f: R→{x∈R:-1<x<1} defined by f(x)=$\frac{1}{1+|x|}$,x∈ $R$is one and onto function.

5. On the set {1,2,3,4,5,6}, a binary operation *is defined as:

a*b=$\begin{cases} a+b \\ a+b-7 \end{cases}$ if a+b<7 if a+b≥ 7

write the operation table of the operation * and prove that zero is the identity for this operation and each element a≠0 of the set is invertible with 7-a being the inverse of a.

6.Let A=N x N and * be the binary operation on A defined by (a, b)*(c, d)=(a + c, b + d). Show that * is commutative and associative. Find the identity element for *on A, if any.

7.Let A=Q x Q ,where Q is the set of all rational numbers, and *be a binary operation on A defined by (a, b)*(c, d)=(ac, b+ ad) for (a, b),(c, d)∈A. Then find

(i). The identity element of*in A.

(ii). Invertible elements of A, and hence write the inverse of elements(5,3) And ($\frac{1}{2}$, 4)

8. Let f:N→N be defined by f(n)=$\begin{cases} n-1 \\ n+1 \end{cases}\begin{cases} ifnisodd \\ ifniseven \end{cases}$for all n∈N. State whether the function f is bijective , Justify your answer. If it is invertible then find inverse.

9. If the function f:R→R is given by f(x)=x^2+2 and g:R→R is given by g(x)=$\frac{x}{x-1}$ x≠1, then find fog and gof and hence find fog(2) and gof(-3).

10.Show that each of the relation R in the set A={5,6,7,8,9}, given by (i) R={(a ,b):|a-b| is a multiple of 2}. Find the set of all elements related to 6 in each case. (4)

11. Let * be a binary operation on Q_0(set of non zero rational numbers) defined by a*b=$\frac{ab}{4}$ (a,b∈ Q_0) Then find

(i) Identity element in Q_0

(ii) Inverse of an element in Q_0

12.Check that each of the relation R in the set A={1,2,3,4,5,6}, given by (i)R={(a, b):b=a+1} is a reflexive, symmetric or transitive.

13. Show that the function f:R→ {x∈ R : -1<x<1} defined by f(x)=$\frac{1}{1+|x|}$,x∈R is invertible. Find f^{-1}

14. On the set {0,1,2,3,4,5} , a binary operation *is defined as:

a * b=$\begin{cases} a+b \\ a+b-6 \end{cases}\begin{cases} if a+b<6 \\ if a+b\geq 6 \end{cases}$

For the binary operation defined below, determine whether *is commutative or associative on R-{-1} write the operation table of the operation *and prove that zero is the identity for this operation and each element 'a' of the set is invertible with (6,-a) being the inverse of 'a'.

15 .Prove that the Greatest Integer function f: R→R, given by f(x)= x-[x], is neither one-one nor onto , where [x] denotes the greatest integer less than or equal to x.

16. Show that the function f: R→R defined by f(x) = ax-b, x∈ $R$is bijective.

17. Show that the function f: R→R defined by f(x) = x^3+x, x∈ $R$is bijective.

18. Consider the binary operation * : R x R→R and O:R x R→R defined by a*b=|a-b| and a O b =a for all a , b∈R .Show that *is commutative but not associative ,O is associative but not commutative.

19. For the binary operation defined below, determine whether * is commutative or associative on R-{-1} defined a*b=$\frac{a}{b+1}$.

CHAPTER: 3

MATRICES

Level 1:

1. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & -1 & 3 \end{bmatrix}$ = A + $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A
2. If $\begin{pmatrix} cos\frac{2\pi}{7} & -sin\frac{2\pi}{7} \\ sin\frac{2\pi}{7} & cos\frac{2\pi}{7} \end{pmatrix}^K$ = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then find the least positive integral of K.
3. If matrix A = [1 2 3], then write AA' where A' is transpose of matrix.

4. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$ then find the value of x, y

5. If $[a_{ij}] = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{pmatrix}$ and $b_{ij} = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{pmatrix}$ then find $a_{22} + b_{21}$

6. If A = $\begin{bmatrix} cosa & -sina \\ sina & cosa \end{bmatrix}$, then for what value of a is A is identity matrix

7. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$ then find the value of k

8. From the following matrix equation find the value of x: $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

9. Find the value of x+y from the following equation:

   $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

10. If A' = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then find A' – B'

11. If A = $\begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$ then show that A – A' is skew symmetric matrix where A' is the transpose of matrix A

12. Express the following matrix as the sum of symmetric and skew symmetric matrix and verify the result

    $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

13. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find the values of a and b.

14. Find the values of x,y,z, w from the given matrix $\begin{bmatrix} 2x-3y & 3x-y \\ 4 & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$

    (ii) $\begin{bmatrix} 2x-3y & z-w & 3 \\ 1 & x+4y & 3z+4y \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$

15. Find the values of X ,Y from the following:

    $\begin{bmatrix} x-y & 2 & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$

16. If A = diag[2,9,4] B = [-3,7,6] find A +B,A-B,7A+2B, and 9A-11B.

17. Find the order of AB if the order of A and B are respectively (i)2x2 and 2x3(ii)4x1 and 1x 3 (iii)4x4 and 4x1

18. Find A B If A = $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ B = $\begin{bmatrix} i & -1 \\ -1 & -i \end{bmatrix}$

19. If A = $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ B = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ C = $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ show that(I) $A^2 = B^2 = C^2 = -I_2$ (ii) AB =-BA=-C

20. If A = $\begin{bmatrix} -4 & 1 & 9 \\ 3 & 2 & 8 \\ 7 & 5 & 11 \end{bmatrix}$ then show that $AI_3 = A = I_3A$

21. If A = $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ show that AB = A and BA =B

22. If A = $\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, verify that (AB)' = B' A'

23. Define a symmetric matrix and a skew symmetric matrix. Prove that for A = $\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$,

    A +A' is a symmetric matrix.

24. Express the following matrix as sum of symmetric and skew symmetric

(i)A = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

(ii)A = $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$

25. If a matrix has 12 elements what are the possible orders it can have? What if it has 11 elements.

Level 2:

1. If A = $\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$and A^2 = I then find the value of α^2+$\beta\gamma$
2. If A is a square matrix such that A^2=A then show that $(I+A)^3$=7A+I.
3. Find the product of $[x \quad y \quad z]\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$
4. If A = $\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and A^2 –kA +2I =O where I is the identity matrix of order 2, find the value of k.
5. If A = $\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ show that A^{-1}= $\frac{1}{19}$ A
6. Construct a matrix A =$[a_{ij}]_{2x2}$ whose elements a_{ij} are given by a_{ij} = e^{2ix}sinjx.
7. Construct a 2x2 matrix A = $[a_{ij}]$ whose elements are given by aij =$\begin{cases} i-j\,;if\ i\geq j\\ i+j\,;if\ i<j\end{cases}$
8. Find a and b if $\left\{3\begin{bmatrix}2 & 1 & -3\\ 1 & 4 & 2\end{bmatrix}-2\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\ 0\\ -1\end{bmatrix}$ = $\begin{bmatrix}a\\ b\end{bmatrix}$
9. Find the integral value of x , if $[x \quad 4 \quad -1]\begin{bmatrix}2 & 1 & -1\\ 1 & 0 & 0\\ 2 & 2 & 4\end{bmatrix}[x \quad 4 \quad -1]^T$ = O
10. If the matrix $\begin{bmatrix}0 & 6-5x\\ x^2 & x+3\end{bmatrix}$ is symmetric. Find the value of x.
11. If the matrix $\begin{bmatrix}-2 & x-y & 5\\ 1 & 0 & 4\\ x+y & z & 7\end{bmatrix}$ is symmetric find the value of x,y,z
12. If A = $\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ a & b & c\end{pmatrix}$ then show that A^3 = aI +bA+cA^2, where I is the identity matrix of 3.
13. Find x, if $[x \quad -5 \quad -1]\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}$ = O
14. If A = $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 5\end{bmatrix}$ B = $\begin{bmatrix}0 & 2 & 4\\ 2 & 4 & 6\\ 4 & 6 & 8\end{bmatrix}$ express 3A -2B as the sum of symmetric and skew symmetric matrices.
15. If A^2=A for A =$\begin{bmatrix}-1 & b\\ -b & 2\end{bmatrix}$ then the value of b is
16. If A and B are symmetric matrices of same order , show that AB-BA is a skew symmetric matrix.
17. If A = $\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ B =$\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$ verify that $(A+B)^2 \neq$ A^2+2AB+B^2
18. Evaluate [a b]$\begin{bmatrix}c\\ d\end{bmatrix}$ + $[a \quad b \quad c \quad d]\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}$
19. Evaluate $\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{pmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{pmatrix}$
20. For A = $\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}$ show that A^n = $\begin{bmatrix}1 & na\\ 0 & 1\end{bmatrix}$ for all positive integers of n.
21. If f(x) = x^3+4x^2-x find f(A) where A =$\begin{bmatrix}0 & 1 & 2\\ 2 & -3 & 0\\ 1 & -1 & 0\end{bmatrix}$
22. Find the product of matrices A = $\begin{bmatrix}cos^2\alpha & cos\alpha sin\alpha\\ cos\alpha sin\alpha & sin^2\alpha\end{bmatrix}$ and B = $\begin{bmatrix}cos^2\beta & cos\beta sin\beta\\ cos\beta sin\beta & sin^2\beta\end{bmatrix}$ and show that AB is null matrix if α and β differ by an odd multiple of $\frac{\pi}{2}$
23. If $A_{n\theta}$ = $\begin{bmatrix}cosn\theta & sinn\theta\\ -sinn\theta & cosn\theta\end{bmatrix}$ then using mathematical induction prove that $(A_\theta)^n$ = $A_{n\theta}$
24. If A is 3 x 4 matrix and B is matrix such that A'B and BA' are both defined , then find the order of matrix B.
25. Define a symmetric matrix and a skew symmetric matrix. Prove that for A = $\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$,A +A' is a symmetric matrix.

26. Find the value of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ obeys the law A'A =I

27. Find the values of a and b such that the matrix A = $\frac{1}{3}\begin{bmatrix} a & -2 & -2 \\ -2 & -1 & b \\ -2 & b & -1 \end{bmatrix}$satisfies AA' = I

28. Construct a 2 X 2 matrix A =$[a_{ij}]$ whose elements are given by a_{ij} = $\frac{(i+2j)^2)}{2}$

29. Find the matrix X such that 2A +B +X = 0 where A = $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

30. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find k so that A^2 = KA -2I.

31. If A = $\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, verify that (AB)' = B' A'

32. For the matrices A ,B and C given by A= $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 5 & 7 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ and C = $\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ - & 3 & 5 \end{bmatrix}$, then show that (i) A^2= I (II) B^4 = O (III) C^2=C

33. If A = $\begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$ show that A^2 = $\begin{matrix} cos2\alpha & sin2\alpha \\ -sin2\alpha & cos2\alpha \end{matrix}$

34. If A =$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ find A^2 and A^3

35. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix}$ = $\begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ obtain the values of a,b,c,x,y,z

36. If A = $\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$ and B =$\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ then find the matrix C such that A+B+C is zero matrix.

37. If A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ show that A^2 -5A +4I =O hence find A^{-1}

38. Show that A =$\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation X^2- 6X+17 =0Hence find A^{-1}

39. Find A^2-5A+6I , if A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

40. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find k so that A^2 = KA-2I

41. If A =$\begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$ then show that A – A' is skew symmetric matrix where A' is the transpose of matrix A

42. Let A =$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, B =$\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ find a matrix D such that CD – AB =O

43. Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation A^2-4A+I = O where I is 2 x 2 identity matrix and O is the 2 x2 zero matrix using this equation find A^{-1}

44. Prove that the sum of two skew symmetric matrices is a skew – symmetric matrix.

45. Find the value matrix X so that x$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ = $\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

46. For what value of x, the matrix A =$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew – symmetric matrix.

47. If matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A^2 = KA, then write the value of k.

48. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}$ = 0

49. For the following matrix verify (AB)'= B' A'

A=$\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, B=$\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

50. In a parliament election, a political party hired a public relations firm to promote its candidates in three ways – telephone, house calls and letters . the cost per conduct is given in matrix as A = $\begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$

The number of contacts of each type made in two cities X and Y is given in the matrix B as B= $\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$ find the total amount spend by the party in the two cities

51. A manufacture produces three products x,y,z which he tells in two markets annual sets are indicate in the table

MARKET	PRODUCTS		
	X	Y	Z
I	10000	2000	18000
II	6000	20000	8000

If unit rate of x,y,z are 2.50/- ,1.50/-, 1.00/- respectively, then find the total revenue in each market , using matrices.

52. To promote the making of toilets for women , an organisation tried to generate awareness through (i)house calls(ii)letters(iii) announcements. The cost for each mode per attempt is given below (i)50/- (ii)20/-(iii)40/-the number of attempts made in three villages x,y,z are given below

	(I)	(II)	(III)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages separately ,using matrices.

53. Three schools A,B,C want to award their selected students for three values of honesty ,regularity and hard work. Each school decided to award a sum of 2500/-,3100/-5100/- per student for the respective values. The number of students to be awarded by the three schools is given below in the table:

VALUES	A	B	C
HONESTY	3	4	6
REGULARITY	4	5	2
HARDWORK	6	3	4

Find the total money given in awards by the three schools seperately using matrices

54. Three schools x,y,zorganised a fete for collecting funds for flood victims in which they sold hand-held fans and toys from recycled material the sale price for each being 25/-100/-and 50/- respectively. The following table shows the number of articles of each type sold:

SCHOOL ARTICLE	X	Y	Z
HAND –HELD FANS	30	40	35
MATS	12	15	20
TOYS	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the funds collected.

Level 3:

1. If A and B are skew symmetric matrices of same order , prove that

   (i)AB+BA is a symmetric matrix.
   (ii)AB-BA is a skew symmetric matrix

2. If A = $\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$ prove that A^n = $\begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$ for all positive integers n.
3. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ prove that A^n = $\begin{bmatrix} 1+2n & -4n \\ n & -1-2n \end{bmatrix}$ for all positive integers.
4. If A = $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ prove that A^n = $\begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$ for all positive integers
5. If A = $\begin{bmatrix} cosa+sina & \sqrt{2}sina \\ -\sqrt{2}sina & cosa-sina \end{bmatrix}$, prove that A^n = $\begin{bmatrix} cosna+sinna & \sqrt{2}sinna \\ -\sqrt{2}sinna & cosna-sinna \end{bmatrix}$ for all n ∈ N.
6. If A = diag[a,b,c] prove that A^n = $[a^n,b^n,c^n]$ for all positive integers n.
7. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ prove that A^n = $\begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ for all positive integers of n.
8. If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ a≠1 , prove by mathematical induction that A^n = $\begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$for all positive integers.
9. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ prove by induction that (a I + b A) =a^nI+na^{n-1}bA, where I is the unity matrix of 2 and n is a positive integer.
10. By using elementary operations find the inverse, if exits, of the following: (i). A = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ (ii) A = $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ (iii) A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$
11. Using elementary row operation find the inverse of the matrix A =$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$
12. Using elementary transformation find the inverse of A = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$
13. Show that $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$
14. If A = $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ then prove that A^n = $\begin{bmatrix} cosn\theta & sinn\theta \\ -sinn\theta & cosn\theta \end{bmatrix}$n∈ N
15. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that A^n = $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ n∈ N
16. Prove the following by the principle of mathematical induction:

    If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then A^n= $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n

17. A former posses 30 acre of land cultivated that must be cultivated in two different mode of cultivations organic and inorganic .the yield for organic and inorganic system of cultivation is 11quin/acre an 14 quin/acre respectively . using matrix method determine how to ivied 30 acre of land among two modes of cultivation to obtained yield 390 quintals.
18. Two schools A and B decided to award prices to their students for three values of honesty (x) punctuality(y) and obedience (z).School A decided to award a total of 11000 for the three values to 5,4, and 3 students respectively ,while school B decided to award of 10700 for the three values to 4,3,and 5 students respectively .If all the three prizes together amount to 2700, then (i) represent the above situation by the matrix equation and form linear equations using matrix multiplication(ii)find the values of x,y,z.

19. For well being of an orphanage three trust A,B,C has donated 10%,15% and 20% of their total fund 2,00,000 , 3,00,000 , and 5,00,000 resp. Using matrix multiplication find the total amount of money received by orphanage by three trust.
20. To rise the amount for an orphanage, students of three schools A,B,C organised an exhibition in their locality ,where they sold paper bags, scarp-books and pastel sheets made by them using recycle paper at the rate of 20/- 15/- and 5/- per unit respectively school A sold 25 papers 12 scarp books and 34 pastel sheets .School B sold 22

paperbags ,15 scarp books 28 postal sheets while school C sold 26 paper bags ,18 scarpbooks 36 pastel sheets . using matrices find the total amount raise by each school
21. A whole sale shop has five dozen t-shirts with BE DISIPLINE printed, three dozen T-shirts BE PUNCTUAL printed and six dozen T-shirts with BE HONEST printed. The cost of each T-shirts is 600/- ,700/- 800/-. All these items are sold in one day. Find the total amount of money collected by shop by using matrix multiplication .which shirt you would like to buy and why?
22. A trust has fund 50000/-that is to be invested in two type of bonds . The first bond pays 10%pa intrest which will be given to adult education and second bond pays 12% pa intrest which will be given to the financial benfits of the trust . Using matrix multiplication ,determine how to divide 50000 amoug two types of bonds if the trust fund obtain the annual intrest of 1800/-.

CHAPTER -13
PROBABILITY

SHORT ANSWER TYPE

1. If P(A) =$\frac{1}{2}$ P(B) = 0 , then find P(A/B).
2. Write the probabilty of even prime number on each die, when a pair of die is rolled.
3. Two independent events A and B are given such that P(A) = 0.3 and P(B) = 0.6, find P(A and not B)
4. A fair coin and unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event 3 on the die'. Check whether A and B are independent events or not.
5. If X ha sa binomial distribution B(4 , $\frac{1}{3}$), then write P(x=1).
6. A four digit number is formed using the digits 1,2,3,5 with no repetitions .find the propability that the number is divisible by 5.
7. Let A and B be the two events. If P(A) = 0.2,P(B)= 0.4,P(A∪B)= 0.6, then find P($\frac{A}{B}$).

8. A random variable x has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2+1$

Determine :(i)k (ii) P(X<3) (iii) P(X>6) (iv)P(0<X<3)

9. How many times must a man toss a fair coin , so that the probability of having atleast one head is more that 80%?
10. A problem in mathematics id given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, what is the probability that the(i) problem is solved?(ii)exactly one of them will solve it?
11. X is taking up subjects, mathematics,physics and chemistry in the examination. His probabilities of getting A in these subjects are 0.2 , 0.3 , and 0.5 respectively. Find the probability that he gets (i)grade A in all subjects (ii) grade A in no subject (iii)grade A in two subjects.
12. Find the probability of atmost two tails or at least two heads in a toss of three coins.
13. Assume that each born child is eually likely to be a boy or girl. If a family has two children, what is the conditional probablility that both are girls given that
    (i) The youngest is a girl (ii)At least one is a girl
14. An experiment succeeds thrice as often as it fails. Find the probability that the next five trails, there will be atleast 3 successes.

15. Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II.
16. A bag contains 1 white and 6 reed balls, and second ball contains 4 white and 3 red balls. One of the bags is picked up at random and a ball is randomly drawn from it, and is found to be white in colour. Find the probability that the drawn ball was from the first bag.
17. Three cards are drawn at random from a pack of 52 playing cards. Find the probability distribution of the number of aces.
18. Three cards are drawn from well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence , find the mean of the distribution.

19. If two dice are rolled 12 times, obtain the mean and variance of the distribution of success, if getting a total greater than 4 is considered as success.
20. A die is thrown 6 times. If 'getting an odd number' is success, what is the probability of(i) 5 success? (ii)at least 5 success (iii)at most 5 success?
21. Six coins are tossed simultaniously, find the probability of getting
    (i) 3 heads
    (ii) No heads
    (iii) At least one head.
22. Two cards are drawn simultaniously (with out replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.
23. A pair of dice is thrown 4 times. If getting a doublet is considerd a success, find the probability distribution of number of success and hence find its mean.
24. A and B throw a pair of dice turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show thatb the probability of A getting prize is $\frac{9}{17}$.
25. A die is thrown again and again untill three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of die.
26. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it is $\frac{2}{5}$. If each one of A and B shoots at the target , what is the probability that (i) the target is hit (ii) exactly one of them hits the target.
27. An experiment succeeds twice often as it fails. Find the probability that in the next six trails their will be at least 4 successes.
28. Find the probablity of throwin at most 2 sixes in 6 throws of a single die.

29. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random(with out replacement) and are found to be all spades. Find the probability of lost card being a spade.
30. Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferrd from bag I to bag II and then a ball is drawn from bag II. The ball is so drawn is to be red in colour. Find the probability that the transferd ball is black.
31. A girl throws a die. If she gets 5 or 6 , she tosses a coin three times and note the number of heads. If she gets 1,2,3 or 4, she tosses a coin two times and notes the number of heads obtained. If she obtain exactly two heads, what is the probability that she threw 1,2,3 or 4 with the die?
32. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.
33. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

LONG ANSWER TYPE

34. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X.Also, find the mean and variance of the distribution.

35. In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides.
36. Two the numbers are selected at random (without replacement) from first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Find the mean and variance of this distribution.
37. Two numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X.Also, find the mean and variance of the distribution.
38. Three bag conatin balls as shown in the table below:

bag	Number of white balls	Number of black balls	Number of red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is choosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from IIIbag.

39. Three bag conatin balls as shown in the table below:

bag	Number of white balls	Number of black balls	Number of red balls
I	2	1	3
II	4	2	1
III	5	4	3

A bag is choosen at random and two balls are drawn from it. They happen to be black and red. What is the probability that they come from I bag.

40. There arte three coins. One is two headed coin(having head on both faces), another is a biased coin that comes up head 75% of the times and the third is unbiased coin. One of the three coins is choosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?
41. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement .find the probability distribution of number of defective bulbs , hence find the mean of distribution.
42. From a lot of 10 bulbs , which include3 defectives , a sample of 2 bulbs is drawn at random. Find the probability distribution of number of defective bulbs.
43. There are two bags bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I.
44. A bag contain 4 balls. two balls are drawn at random and are found to be white. What is the probability that all balls are white.
45. An urn contains 4 white and 3 red balls. Let x be the number of red balls in a random draw of three balls find the mean and variance of x.
46. Three machines E1,E2,E3 in certain factory produce 50%,25%and 25% respectively, of the total daily output of electric tubes. It is known that 4% of tube produced on each of machinesE1 and E2 arev defective and that 5% of those produced on E3, are defective. If one tube is picked up at random from a day’s production, calculate the probability that it id defective.

47. The probability that a student entering a university will graduate is 0.4. find the probility that out of 3 students of the university:

(i) none will graduate

(ii) only one will graduate

(iii) All will graduate

48. On a multiple choice examination with three possible answers(out of which only one is correct)for each of the five questions, what is the probability that a candidate would get four or more correct answers just by gussing?
49. In a game a man win a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.
50. A speak truth in 60% of the case, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speak truth in more number of cases than A.
51. A student is given a test with 8 items of true – false type. If he gots 6 or more items correct,he is declared pass. Given that he guesses the answer to each item, compute the probability that he will pass in the test.
52. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is acually a number greater than 4.
53. In a certain college, 4% of boys and 1% of girls are taller than 1.75 meters. Futher more , 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 meters. Find the probability that the selected student is a girl.
54. A factory has two machines A and B. past record shows that machine A produced 60% of the items of the output and machine B produced 40% of the items. Further, 2% of the items prodced by machine B were defective. All the items are put in to one stock pile and then one item is choosen at random from this and is found to be defective. What is the probability that it was produced by the machine B?
55. Two groups are competing for the position on the board of directors of a corporation. The probabilities that first and second group will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probabilty that the new product was introduced by the second group.
56. If each element in the second order determinant is either 0 or 1, what is the probability that value of determinant is positive?(assume that the individual entries of the determinant are choosen independently, each value assumed with probability $\frac{1}{2}$.
57. Let d_1,d_2,d_3 be the three mutually exclusive diseases. Let S={s_1,s_2,s_3,…s_6}be the set of observable symptoms of these diseases. For example, s_1 is the shortness of breath, s_2 is the loss of weight and s_3 is a fatigue, etc. suppose a random sample of 10,000 patients contain 3200 patients with disease d_1,and 3500 with disease d_2, and 3000 with disease d_3 show the symptom S. knowing that the patient has symptom S, the doctor wishes to determine the patient illness. On the basis of the information, what should the doctor conclude?
58. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X=x) = \begin{cases} 0.1, & if x = 0 \\ kx\ , & if x = 1\ or\ 2 \\ k(5-x), & if x = 3\ or\ 4 \\ 0, & otherwise \end{cases}$$

(a). find the alue of k.

(b). what is the probability that you study (i) at least two hours?(ii) exactly two hours?(iii) at most two hours ?

59. P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact? Do you think when they agree, means both are speaking truth?
60. A speaks truth in 75% of the case, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? Do you think that statement of B is true?
61. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of distribution also. Write one more value which is expected from a well trained scout.

62. Assume that patient having a heart attack is 40%. Assuming that meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a couce of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.
63. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is acually a six. Do you also agree that the value of truthfulness leads to more respect in the society?
64. Of the students in the school it is known that 30% have 100% attendence and 70% students are irregular. Previous year results report that 70% of all the students who have 100% attendence attain A garde and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is choosen at random from the school and he has A grade. What is the probability that student has 100% attendance?
    (i). write any two values reflected in this question.
    (ii). Is regularity required only in school? Justify your answer.
65. In shop A, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop B , 50 tin pure ghee and 60 tin adulterated ghee are there. One tin of ghee is purchased from one of the shops randomly and it is found to be adulterated. Find the probability that it is purchased from shop B.
    (i). how is adulteration is dangerous for humanity?
    (ii). What can you do againest adulteration?
66. In a self assessment survey 60% persons claimed that they never indulged incorreption, 40% persons claimed that they always spoke the truth and 20 % said that they neither indulged in corruption nor told lies.
    A person is selected at random out of this group.
    (i). what is the probability that the person is either correpț or tells lie?
    (ii). If a person never indulged in corruption, find the probability that she/he speaks the truth.
    (iii). If a person always speak the truth then find the probability that she/he claims to have never indulged in corruption.
67. In a village there are three mohallas A,B,C. In A, 60% persons believe in honesty, while in B, 70% and in C, 80%. A person is selected at random from village and found, he is honest. Find the probability that he belongs to mohalla B.
    Is a honest person free from corruption? If yes justify your answer.
68. A person want to construct a hospital ina village for welfare. The probabilities are 0.40 that some bad elements oppose this work, 0.80 that the hospital will be completed if there is no any oppose of any bad elements and 0.30 that the hospital will be completed if bad elements will oppose. Determine the probability that the construction of hospital will be completed.
69. In a class XII, it is found that 60% are hostelier and 40% are day scholors. According to result of first term test, it is found that 30% of hosteliersattandistintion and 20% of day scholars attain distintion marks. Now one student is choosen at random fro the classXII and he has distintion marks. What is the probability that the student is a hostelier. Is hostelier better than day scholor? Give your opinion with justification.
70. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer $\frac{1}{4}$be the probability that he guesses and$\frac{1}{4}$ that he copies it. Assuming that a student , who copies the answer, will be correct has the probability $\frac{3}{4}$, what is the probability that the student knows the answer, given that he answered it correctly?
71. The probability of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{7}{5}$ respectively. Assuming that the events , A coming in time and B coming in time are independent, find the probability of only one of them coming to school in time.
72. In a hockey match both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternatively and the decided that the team, whose

73. captain gets a six first, will be declared the winner. If the captain of the team A was asked to start, find their respective probabilities of winning the match and state weather the decision of the referee was fair or not.
74. 4 cards are drawn from a well-shuffled deck of 52 cards.What is the probability of obtaining 3 diamond cards?
75. If A and B are two events such that P(A)=0.42, p(B)=0.48 and P(A∩B)=0.16, then find P(AUB).
76. There are 25 tickets bearing numbers from 1 to 25.One ticket is drawn at random.Find the probability that the number on it is a multiple of 5 or 6.
77. Given that, the two numbers appearing on throwing two dice are different.Find the probability of the events the sum of numbers on the dice is 4'.
78. Events E and F are such that P (not E or not F)=0.25.State whether E and F are mutually exclusive.
79. Ten coins are tossed.What is the probability of getting atleast 8 heads?
80. A lot of 100 watches is known to have 10 defective watches.If 8 watches (one –by-one with replacement) at random,What is the probability that there will be atleast one defective watch?
81. Four cards are successively drawn without replacement from a deck of 52 playing cards.What is the probability that all the four cards are kings?
82. If P(A)=6/11,P(B)=5/11 and P(AUB)=7/11,then find P(A∩B).
83. Compute P(A/B),if P(B)=0.5 and P(A∩B)=0.3.
84. The probability that a student will pass the final examination of both English and hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing English examination is 0.75, then what is the probability of passing the Hindi examination?
85. I E and F events , such that P(E)=1/4,P(F)=1/2 and P(E and F)=1/8,then find
    (i) P(E orF).
    (ii) P(not E and not F).
86. The letter of the word 'SOCIETY' are placed at random in a row. What is the probability that the three vowels come together?
87. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared atleast once?
88. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.
89. The probability of simultaneous occurrence of atleast one of two events A and B is ρ. If the probability that exactly one of A,B occurs is q, then prove that P(A')+P(B')=2-2ρ+q.
90. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective.If one bulb is picked up at random, determine the probability of its being defective, if it is red.
91. Three bags contain number of red and white balls as follows

    Bag I : 3 red balls

    Bag II : 2 red balls and 1 white ball

    Bag II : 3 white balls

    The probability that bag I will be chosen and ball is selected from it is i/6, i=1,2,3. What is the probability that
    (i) A red ball will be selected?
    (ii) A white ball is selected?
92. A shopekeeper sells three types of flower seed A1,,A2 and A3.They are sold as a mixture, where the proportions are 4:4:2, respectively. The germination rates of the three types of seeds are 45%,60% and 35%. Calculate the probability
    (i) Of a randomly chosen seed to germinate.
    (ii) That it will not germinate , given that the seed is of type A3.
    (iii) That it is of the type A2, given that a randomly chosen seed does not germinate.
93. Two dice are thrown ,find the probability of getting an odd number on the first die and a multiple of 3 on the othe die.
94. The probability of A hitting a target is 4/5 and that of B hitting is 2/3. They both fire at the target.Find the probability that atleast one of them will hit the target.
95. Probability of solving specific problems independently by A and B are ½ and 1/3, respectively.If both try to solve the problem independently.Find the probability that

(i) The problem is solved

(ii) Exactly one of them solves the problem

96. Three fair coins are tossed .Find the probability that the outcomes are all tails, if one of the coin shows a tail.
97. A die tossed thrice. Find the probability of getting an odd number atleast once.
98. The probability of A ,B and C solving a problem are ½,1/3 and ¼,respectively. If the problem is attempted by all simultaneously.Then,find the probability of exactly one of them solve it.

99. X is taking up subjects: Mathematics ,Physics and Chemistry in the examination . The probability of getting grade A in these subjects are 0.2,0.3 and 0.5 respectively.Find the probability that he gets

(i) Grade A in all subjects.

(ii) Grade A in no subjects.

(iii) Grade A in two subjects.

100. A coin hit target 4 times out of 5 times , B can hit the target 3 times out of 4 times and C can hit 2 times out of 3 times. They fire simultaneously. Find the probability

(i) Any two out of A,B and C will hit target.

(ii) None of them will hit target.

101. Two dice are thrown together .Let A be the event 'getting 6 on the first die' and B be the event 'getting 2 on the second die'. Are the events A and B independent?
102. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement . What is the probability that both drown balls are black?
103. Two dice are thrown together and thr total score is noted.The events E,F and G are 'a total of 4', 'a total of 9 or more' and 'a total divisible by 5', respectively. Calculate P(E),P(F) and decide which pairs of events, if any, are independent?
104. Three cards are drown successively without replacement from a pack of 52 well-shuffled cards. What is the probability that first two cards are kings and third card drawn is an ace?
105. Three coins are tossed simultaneously. Consider the event E 'three heads or three tails', F 'atleast two heads' and G 'atmost two heads'. Of the pair (E,F), (E,G) and (F,G), which are independent ? which are dependent?
106. There are three coins , one is a two headed coin (having head on both faces),another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is choosen at random and tossed, it shows heads.What is the probability that it was the two headed coin?
107. A factory has two machines A and B. Past records that machines A produced 60% of the item of output and machines B produced 40% of the item. Further, 2% of item produced by machine A and 1% produced by machine B were defective. All the items are put into one stock pile and then one item is choosen at random from this and found to be defective.What is the probability that it was produced by machine B?
108. Bag A contains 3 white and 2 red balls.Bag B contains 4 white and 5 red balls.One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from bag B.
109. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motor cycle is 0.02. An insured vehicle met with an accident .Find the probability that accident vehicle was a motor cycle.
110. A company has two plants to manufacture motorcycle . 70% motorcycles are manufactured at the plant, while 30% are manufacture at second plant. At the first plant, 80% motorcycles are rated of standerdquality , while at the second plant, 90% are rated of standard quality. A motorcycle, randomly picked up, is found to be standard quality. Find the probability that it has come out from the second plant.
111. A factory has three machines I,II and III which produces 30%,50% and 20% respectively of the total items of the same variety. Out of them 2%,5% and 3% respectively are found to be defective An item is picked up at random and found to be defective. Find the probability that it is produced by machine III.
112. A card from a pack of 52 cards is lost. From the remaining cards of the pack ,two cards are drawn and are found to be both diamonds. Find the probability of lost card being a diamond.

113. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. the probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB.A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB?

114. Three machines E1,E2,E3 in a certain factory produce 50%,25% and 25% respectively of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E1 and E2 are defective and that 5% of those produced on E3 are defective .If one tube is picked up random from a day's production,calculate the probability that it is defective.

115. There are two bags,one of which contains 3 black and 4 white balls , while the other contains 4 black and 3 white balls .A die is thrown. If it shows up 1 or 3 , a ball is taken from the first bag; but it shows up any other number , a ball is chosen from the second bag. F the probability of choosing a black ball.

116. There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white .Find the probability that the ball drawn was from the second urn.

117. A doctor is to visit a patient .From the past experience , it is known that the probability that he will come by train ,bus,scooter or by other means of transport, are respectively 3/10, 1/5, 1/10 and 2/5. The probability that he will be late , are 1/4, 1/3 and 1/12, if he comes by train , bus and scooter respectively ,but if he comes by other means of transport , then he will not be late . When he arrives, he is late .What is the probability that he comes by train?

118. A class has 15 students, whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean , variance and standard deviation (SD) of X.

119. An urn contains 6 red and 3 black balls .Two balls are randomly drawn . Let X represents the number of black balls. What are the possible values of X ?

120. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards . Find the probability distribution of numbers of spades.

121. State whether the following is the probability distribution of a variable or not.

122. defective bulbs are mixed up with 7 good ones.

123. 3 bulbs are drawn at random. Find the probability distribution of defective bulbs.

124. Find mean and variance of number of heads in three tossed of a fair coin.

125. A discreat random variable X has the following probability distribution

X	1	2	3	4	5	6	7
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2$+C

Find the value of C . Also find the mean of the distribution.

126. A person plays a game of tossing a coin thrice .For each head, he is given ₹2 by the organizer of the game and for each tail, he has to give ₹ 1.50 to the organizer .Let X denotes the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.

127. Two cards are drawn successively with replacement from a well- shuffled deck of 52 cards. Find the probability distribution of the number of aces.

128. Let X denotes the number of hours you study during a randomly selected school selected school day. The probability that X can take the values X, has the following form , where k is some unknown constant.

$$P(X=x)=\begin{cases} 0.1 & if x = 0 \\ kx if x = 1 \text{ or } 2 \\ k(5-x) if x = 3 \text{ or } 4 \\ 0 & otherwise \end{cases}$$

(i) Find the value of k.

(ii) What is the probability that you study (a) atleast two hours?(b) exactly two hours? (c) atmost two hours?

129. The probability distribution of a random variable x is given as under

$$P(X=x)=\begin{cases} kx^2 \; for x = 1,2,3 \\ 2kx for x = 4,5,6 \\ 0 \quad otherwise \end{cases}$$

Where k is a constand. Calculate

(i)E(X) (ii) E($3X^2$) (iii)P(X ≥4)

130. A die is thrown 6 times .If getting an odd number is a success, what is the probability of 5 success?

131. A bag consisits of 10 balls , each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag. What is the probability that none is marked with the digit 0?

132. Two cards are drawn successively with replacement from a well-suffuled deck of 52 cards.Find the probability distribution of number of aces.

133. A coin is biased , so that head is 3 times as times as likely to occur as tail. If the coin is tossed twice. Find the probability distribution of number of tails.

134. How many times must a man toss a fair coin, so that the probability of having atleast one head is more than 80%?

135. Find the probability that in 10 throws of a fair die, a score which is a multiple of 3 will be obtained in atleast 8 of the throws.

136. If a fair coin is tossed 10 times, find the probability of
(i) Exactly six heads
(ii) Atleast six heads
(iii) Atmost six heads

137. Ten eggs are drawn successively with replacement from a bag containing 10% defective eggs. Find the probability that three is atleast one defective egg.

138. On a multiple choice examination with three possible answers for each of the five question , what is the probability that a candidate would get four or more correct answers just by guessing?

139. A person busy a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is 1/100. What is the probability that he will win a prize
(i) Atleast once?
(ii) Exactly once?
(iii) Atleast twice?

140. Two coins are tossed. What is the probability of coming up two heads if it is known that at least one head comes up.

141. Four cards are drawn from 52 cards with replacement. Find the probability of getting at least 3 aces.

142. A bag contains 5 white and 4 red balls. 2 balls are drawn from the bag. Find the probability that both balls are white.

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143. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade.

144. From a well, shuffled pack of 52 cards, 3 cards are drawn one-by-one with replacement. Find the probability distribution of number of queens.

145. Two dice are thrown and it is known that the first die shows a '6'. Find the probability that the sum of numbers showing on the dice is 7.

146. A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of getting:

(i) exactly 6 successes? *(ii)* at most 6 successes?

147. A bag contains 10 balls each marked with one of the digits 0 to 9. If 4 balls are drawn successively with replacement from the bag, what is the probability that none is marked with digit 0?

148. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Whatisthe probability that(i)all the five cards are spades? (ii) only 3 cards are spades.(iii)none is a spade?

149. Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of kings.

150. Two cards are selected at random from a box which contains five cards numbered 1, 1, 2, 2, and 3. Let X denote the sum and Y the maximum of the two numbers drawn. Find the probability distribution, mean and variance of X and Y.

151. The probability that a bulb produced by a factory will fuse after 100 days of use is 0.05. Find the probability that out of 5 such bulbs

(i) not more than one *(ii)* more than one

will fuse after 100 days of use.

CHAPTER 4

DETERMINANTS

VERY SHORT ANSWER QUESTIONS.

1. Write the value of $\begin{vmatrix} \sin 20^0 & -\cos 20^0 \\ \sin 70^0 & \cos 70 \end{vmatrix}$

2. If A is a square matrix of order 3 such that |A| = λ , then write the value of |-A|

3. Write the value of x_1 the following matrix is singular? $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$

4. Evaluate $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$

5. Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

6. Find the co-factor of a_{12} in the following:

$$\begin{vmatrix} 1 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

7. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, then find the value of x.

8. Write the value of the following determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

9. Write the value of the following determinant

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

10. Find the value of 'x' from the following

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$$

11. Write the value of $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$

12. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of K if |2A| = K|A|.

13. What positive value of 'x' makes the following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

14. What is the value of the following determinant $\Delta = \begin{vmatrix} 4 & a & b+c \\ 4 & b & a+c \\ 4 & c & a+b \end{vmatrix}$

15. Evaluate the determinant; $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

16. Find the minor of the element of second now and third column (a_{23}) in the following determinants

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

17. Evaluate $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$

18. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, then write the positive value of 'x'

19. If $\Delta \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, then write the minor element a_{23}

20. Write the value of the following determinant:

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

21. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix}$, then write the value of 'x'

22. If Aij is the co-factor of the element aij of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the

value of $a_{32} A_{32}$.

23. If A = $\begin{vmatrix} 3 & 10 \\ 2 & 7 \end{vmatrix}$, then write A^{-1}; $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ find A^{-1}.

24. Write the value of $\Delta \begin{bmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & 3 \end{bmatrix}$

25. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number 'n' find the value of $\text{Det}\left(\text{A}^n\right)$.

26. If A is a square matrix of order 3 such that |AdjA| = 64, find $\left|\text{A}^1\right|$

27. Find the adjoint of matrix $\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

28. If A = [1 2 3] write A.A^1, where A^1 is transpose of Matrix A.

29. If A is an invertible matrix of 3x3 and |A| = 7, then find $\left|\text{A}^1\right|$

30. For what value x, the matrix $\text{A}=\begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$

31. Find the value of $\begin{bmatrix} \sin A & -\sin B \\ \cos A & \cos B \end{bmatrix}$, where $\text{A}=63^0$ and $\text{B}=27^0$.

32. Find the value of $\begin{vmatrix} \cos A & \sin A \\ -\sin B & \cos B \end{vmatrix}$, where $\text{A} = 75^0$ and $\text{B} = 45^0$

$$\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0 \quad is \quad -(a+b+c)$$

33. If $\begin{vmatrix} \sqrt{6} & x \\ \sqrt{20} & \sqrt{24} \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 5 & 2 \end{vmatrix}$, then write the value of x.

34. If $x \in \mathrm{N}$ and $\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$, then find the value(s) of x.

35. If $x \in \mathrm{I}$ and $\begin{vmatrix} 2x & 3 \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ x & 3 \end{vmatrix}$, then find the values of x.

36. If $x \in R, 0 \le x \le \frac{\pi}{2}, and \begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$ then find the value of x.

37. Find the Integral values of x if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$

38. Show that $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

39. Show that the value of determinant $\begin{vmatrix} 0 & \tan x & 1 \\ 1 & -\sec x & 0 \\ \sec x & 0 & \tan x \end{vmatrix}$ is independent of x.

40. Using co-factors of element of second row, Evaluate $\Delta \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

41. Show that |A| = |B|, where

$$A\begin{vmatrix}1 & -3 & 2\\ 2 & 3 & -1\\ 0 & 5 & 4\end{vmatrix} \text{ and } B=\begin{bmatrix}1 & 2 & 0\\ -3 & 3 & 5\\ 2 & -1 & -4\end{bmatrix}$$

42. With out expanding find the value of

$$i)\begin{vmatrix}a+b & 2a+b & 3a+b\\ 2a+b & 3a+b & 4a+b\\ 4a+b & 5a+b & 6a+b\end{vmatrix}$$

$$ii)\begin{vmatrix}1 & 2 & 4\\ 8 & 16 & 32\\ 64 & 128 & 56\end{vmatrix} \quad iii)\begin{vmatrix}1 & 3 & 5\\ 9 & 7 & 11\\ 13 & 15 & 17\end{vmatrix}$$

$$iv)\begin{vmatrix}x+y & y+z & z+x\\ 2 & 2 & 2\\ z & x & y\end{vmatrix} \quad v)\begin{vmatrix}43 & 1 & 6\\ 35 & 7 & 4\\ 17 & 3 & 2\end{vmatrix}$$

$$vi)\begin{vmatrix}a+b & b-c & c-a\\ x-y & y-z & z-x\\ p-q & q-r & r-p\end{vmatrix}$$

$$vii)\begin{vmatrix}\frac{1}{a} & 1 & bc\\ \frac{1}{b} & 1 & ca\\ \frac{1}{c} & 1 & ab\end{vmatrix} \quad viii)\begin{vmatrix}1 & ab & \frac{1}{a}+\frac{1}{b}\\ 1 & bc & \frac{1}{b}+\frac{1}{c}\\ 1 & ca & \frac{1}{c}+\frac{1}{a}\end{vmatrix}$$

$$ix)\begin{vmatrix}\sin\alpha & \cos\alpha & \cos(\alpha+\delta)\\ \sin\beta & \cos\beta & \cos(\beta+\delta)\\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta)\end{vmatrix} \quad x)\begin{vmatrix}(2^x+2^{-x})^2 & (2x-2^{-x})^2 & 1\\ (3^x+3^{-x})^2 & (3^x-3^{-x})^2 & 1\\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1\end{vmatrix}$$

43. If A is square matrix of order 2 and |A| = 5 find the value of |3A|

44. If A is a square matrix of order 3 and |A| = 4, then write the value of |2A|

45. If A is a matrix of order 3x3 and its determinant is 4, then find |3A|.

46. If A= $\begin{bmatrix} x+4 & 2 \\ 3 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} x & 2x-1 \\ 2 & 5 \end{bmatrix}$ and |AB| = 12, then find the values of x.

47. If A = $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ $and\ \mathrm{A}_1, \mathrm{A}_2, \mathrm{B}_1, \mathrm{B}_2$ denote the co-factors of the corresponding elements $a_1\, a_2\,, b_1\, b_2$ of A respectively, then write the value.

48. Show that one root of the equation

$$\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$$ is –(a+b+c)

50. Using determinants find the equation of the line passing through the

51. If the area of a triangle with vertices (-3, 0) (3, 0) and (o, k) is 9 sq units; find the value of k.

52. If the points (9, 0) (0, b) and (1, 1) are collinear prove that a+b = ab.

53. Find 'k' if the points (k+1, 1) (2k+1, 3) and (2k+2, 2k) are collinear.

54. If $0 < x < \pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$

55. If $0 < x < \pi$ and the matrix $\begin{bmatrix} \cos x & 1 \\ 3 & 4\cos x \end{bmatrix}$ is singular find the values of x.

56. Which of the following matrices are non singular

$$i)\begin{bmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix} ii)\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$$

57. Write ad joint of the matrices;,

$i)\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} ii)\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

58. For what value of k, the matrix $\begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$ has no inverse?

59. Write the inverse of $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

60. If A is a square matrix of order 3 and |A| = 5, then find |adj A|.

61. If A is a non-singular matrix of order 3 such that |adj A| = 64, find |A|.

62. If A = $\begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, find |A adj A|

63. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ find |A adj A|

64. If A is a non-singular matrix, then show that $|A^{-1}| = \frac{1}{|A|}$

65. If A is a matrix such that $A^3 = I$, then show that A is invertible.

66. If A is matrix such that $A^4 = I$ then show that $A^{-1} = A^3$.

67. If A is skew – symmetric Matrix of order 3, then show that |A| = 0.

68. If A = $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ find A(adj A) with out computing adj A.

69. Usmg properties P.T $\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix} = 2(a-b)(b-c)(c-a)(a+b+c)$

70. If a+b+c, and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then using properties of determinants prove that a+b+c = 0

71. Using properties P.T $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

72. ertiesUsing prop, solve for x:

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

73. Using properties of determinants P.T

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x & x(x-1)(x-2 & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$$

74. Using properties of determinant prove that

1. $$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

2. $$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

3. $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2a & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^2$$

4. $$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Using properties PT

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

5. $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

6. $$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

7. $$\begin{vmatrix} x+1 & x+2 & x+9 \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = c$$ where a, b, c are in A.P

8. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

9. $\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

10. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

11. Without expanding show that

$$\begin{vmatrix} co\sec^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \cos ec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$$

12. P.T $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

13. P.T $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$

14. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

15. $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

16. $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & c & q \\ u & v & w \end{vmatrix}$

17. Using properties S.T $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc-a^3-b^3-c^3)$

18. If a, b, c are positive and unequal then show that the following determinant is negative

$$\Delta\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

19. If a, b and c are all positive and distinct, show that $\Delta\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value.

20. Prove the following, Using properties of determinant

$$\begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x^4-1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$$

21. P.T $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

22. P.T $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

23. Using properties of determinants, solve the following for x:

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

24. P.T $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

25. P.T $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

26. P.T $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

27. P. T $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab+bc+ca+abc$

28. P.T $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

29. P.T $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+2b & 10a+6b+3c \end{vmatrix} = a^3$

30. With out expanding Evaluate

$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1 \\ (a^z+a^{-z})^2 & (a^z-a^{-z})^2 & 1 \end{vmatrix}$, where a>o, x, y, z $\in$ R.

31. Find the inverse of matrix A = $\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ and show that a. $A^{-1} = (a^2+bc+1)\text{I-aA},$ also show that $A^{-1} = 10\text{I-A}$, where A= $\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$

32. Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find k if D(k, o) is a point such that the area of ΔABC is 3 sq. units.

33. In a triangle ABC if

$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin c \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin c+\sin{}^2C \end{vmatrix}$ the prove that Δ ABC is an Isosceles triangle.

34. Let F(t) = $\begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}, then find \frac{1t}{t \to 0} \frac{f(t)}{t^2}$

35. S.T $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

36. P.T $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$

37. P.T $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

38. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

39. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

40. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-a-c & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

41. P.T $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

42. With out expanding S.T $\begin{vmatrix} \cos ec^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \cos ec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$

43 If A = $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ $B = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then P.T A + B =0

44. P.T $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$ = x^3

45. $\begin{vmatrix} x & a & a \\ a & x & a \\ \alpha & a & x \end{vmatrix} = (x-a)^2(x+2a)$

46. $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & a^2 \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3+b^3)^2$

47. S.T if $\Delta = \begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \sin 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$, then $\sin\theta = 0 (\text{or}) \frac{1}{2}$

48. If A+B+C = 0, then P.T $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 0$

49. Using properties of determinant

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

50. S.T with out Expanding

$$\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = 0$$

51. S.T $\begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ \sin x & \cos x & \sin y \\ -\cos x & \sin x & \cos y \end{vmatrix}$ is independent of x. only

52. With out expanding S.T

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$$

53. With out Expanding S.T $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

54. If a, b, c in A.P P.T $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

55. Using properties P.T $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

56. P.T $\begin{vmatrix} 1 & 1-p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

57. P.T $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2+px-2q^2)$

58. $\begin{vmatrix} 1 & p/q & \alpha + p/q \\ 1 & r/q & \alpha + r/p \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 2$

59. P. T $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & ca & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

60. S.T $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

61. T $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin^2 x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix} = 2+4\sin 2x$

62. $\begin{vmatrix} 1 & p/q & \alpha + p/q \\ 1 & r/q & \alpha + r/p \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$, ST $P\alpha^2 + 2q\alpha + r = 0$

63. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

64. $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

65. $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$

66. $In\,\Delta ABC, \text{ if } \begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos A+\cos^2 A & \cos B+\cos^2 B & \cos c+\cos^2 c \end{vmatrix} = 0$

The prove tha ABC is an Isosceles triangle.

67.
If A is a square matrix of order 3 such that |adj A| = 64, then find |A|.

68. If $A^2 - A + I = 0$, then find inverse of A

69.
If A is non singular matrix such that

$A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix} \textit{then find} \left(A^1\right)^{-1}$

70. If A and B are non-singluar square matrices of the same order, then write the relation ship

between adj AB, adj A and adj B.

71. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, *find the value of* | adjA |.

72. If A is an invertible matrix of order 3 such that $|AdjA|$ = 64, then find $|A|$.

73. If A is invertible matrix of 3X3 and $|A|$ = 7, Then Find $|A^{-1}|$

74. If 'A' is matrix of order 3x3, Then Find $\left(A^2\right)^{-1}$

75. Write the Ad joint of the following matrix

$\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

76. Write A^{-1} for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

LONG ANSWER QUESTIONS

77. If A = $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then find the value of λ, for which A^{-1} Exists.

78. Find the $(AB)^{-1}$ If A^{-1} = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ = B $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

79. Use Product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ = $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & 2 \end{bmatrix}$ to solve the system of equation x-y+2z=1, 2y-3z=1, 3x-2y+4Z=2,

80. Solve the following system of equations by using matrix method.
$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}$

81. Using matrices solve the following system of equation:
x-y+z = 4, 2x+y-3z = 0, x+y+z = 2

82. If A= $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ then find A^{-1} and hence solve the following system of linear equations.

83. Using Matrices, solve the following system of linear equations:

$$x-y+2z=7, 3x+4y-5z=-5, 2x-y+3z=12$$

84. If A, B are square matrices of the same order then prove that adj (AB) = (adj B) (adj A).

85. A mixture is to be made of three foods A, B, C the three foods A, B, C contain nutrients P, Q, R as shown below.

Food	Ggms per Kg in nutrient		
	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

How to form a mixture which will have 8 gms of P, 5grams of Q and 7 grams of R?

86. If A= $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then verify that $A^3-6A^2+9A-4I=0$ and hence find A^{-1}.